S6 PURE MATHEMATICS 22-10-98

Half-term homework **Show all working clearly**
To be handed in on Monday 2-11-98

1) The first term of a geometric series is 2500 and the fifth term is 4. Given that the common ratio is positive, calculate
a) the seventh term of the series
b) the sum to infinity of the series. **[5]**

2) i) Find $\sum_{r=1}^{100} \frac{2r}{3}$

ii) Find the sum to infinity of $\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \ldots$, giving your answer as a fraction.

Hence or otherwise, find the value of $\sum_{r=1}^{\infty} \frac{k}{10^r}$ in terms of k **[6]**

3) An arithmetic series has first term 24 and common difference -3.
a) Prove that the sum S_n of the first n terms of this series is given by $2S_n = 3n(17 - n)$
b) Given that $S_n = 105$ find the possible values of n. **[8]**

4) The third and sixth terms of a geometric series are 4 and $-\frac{1}{2}$ respectively.
a) Find the first term of this series.
b) Find, to three significant figures, the sum of the first 10 terms of this series. **[7]**

5) Calculate the values of a) $\sum_{r=1}^{100} r$, b) $\sum_{r=1}^{50} 2r$

Hence or otherwise, evaluate the sum $\sum_{r=1}^{50} (2r - 1)$ **[5]**

6) The points *A*, *B* and *C* have coordinates $(-3, 4)$, $(-1, 9)$ and $(7, 8)$ respectively.
a) Calculate the gradient of *AB*.
b) Find an equation of the line which passes through *C* and is parallel to *AB*. **[4]**

7) a) Show that the triangle *PQR* with vertices *P*$(-1, 7)$, *Q*$(8, 19)$ and *R*$(-8, 31)$ is right-angled.
b) Find an equation of the line *PQ*.
c) Calculate the area of ΔPQR. **[6]**

8) The first four terms of an arithmetic progression are 2, $a - b$, $2a + b + 7$ and $a - 3b$ respectively, where a and b are constants. Find a and b and hence determine the sum of the first 30 terms of the progression. **[5]**

9) A line ℓ passes through the point $P(3, k)$, where k is a constant, and is parallel to the line joining the points $A(1, 4)$ and $B(k, 5)$.
a) Find, in terms of k, an equation of ℓ.
b) Show that ℓ passes through the point whose coordinates are $(k + 2, k + 1)$.
c) Given that the angle between the line ℓ and the positive x axis is $45°$, find the possible values of k. **[8]**

10) a) Find, in terms of x and y, the distance between the point $P(x, y)$ and the point $M(3, 7)$.
b) In the same way, find the distance between $P(x, y)$ and the point $N(7, 1)$.
c) Use your answers to form and simplify an equation for the path of *P* if it moves so that the distances *PM* and *PN* are equal. **[8]**

11) ℓ_1 is the line $2x + y = 5$ and ℓ_2 is the line $x + 4y = 6$.
a) Calculate the coordinates of the point of intersection of ℓ_1 and ℓ_2.
b) Calculate in degrees to one decimal place, the angle between ℓ_1 and ℓ_2.
(A sketch diagram might be useful.) **[6]**